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Hardy type inequalities in higher dimensions with explicit estimate of constants

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Abstract

Let Ω be an open set in ℝn such that Ω ≠ ℝn. For 1 ≤ p < ∞, 1 < s < ∞ and δ = dist(x,∂Omega;) we estimate the Hardy constant cp(s, Ω) = sup{||f/δs/p||Lp(Ω): f ∈ C 0 ∞(Ω), ||(▽f)/δ s/p-1||Lp (Ω = 1} and some related quantities. For open sets Ω ⊂ ℝ2 we prove the following bilateral estimates min{2,p} M0(Ω) ≤ cp(2,Ω ≤ 2p (πM0(Ω) + a0)2, a0 = 4.38, where Mo(Ω) is the geometrical parameter denned as the maximum modulus of ring domains in Ω with center on ∂Ω. Since the condition M 0(Ω) ≤ ∞ means the uniformly perfectness of ∂Ω, these estimates give a direct proof of the following Ancona-Pommerenke theorem: C2(2, Ω) is finite if and only if the boundary set ∂Ω is uniformly perfect (see [2], [22] and [40]). Moreover, we obtain the following direct extension of the one dimensional Hardy inequality to the case n ≥ 2: if s > n, then for arbitrary open sets Ω ⊂ ℝn (Ω ≠ ℝn) and any p ∈ [1, ∞) the sharp inequality cp(s, Ω) ≤ p/(s - n) is valid. This gives a solution of a known problem due to J.L.Lewis [31] and A.Wannebo [44]. Estimates of constants in certain other Hardy and Rellich type inequalities are also considered. In particular, we obtain an improved version of a Hardy type inequality by H.Brezis and M.Marcus [13] for convex domains and give its generalizations.

Keywords

Distance to the boundary, Hardy type inequalities, Rellich type inequalities, Uniformly perfect sets